Standard Cosmological models

Cosmology: building representations of the "world"

$\sim$ one of the oldest activity of human kind.

$\rightarrow$ prejudices can be strong (and hidden)

Physical Cosmology...

Simple principles

$+$

Physical laws

- $\longrightarrow$ a consistent picture
- $\longrightarrow$ predictions
- $\longrightarrow$ check/validation by observations

... is a mature science.

$\neq$ having basic principles for the universe

$\rightarrow$ infers physical laws

- "Perfect Cosmological Principle"

  $\rightarrow$ violates energy conservation

- "Tired light theory"

  $\rightarrow$ ν shift is due to some unknown physical process.

However:

- introduction of the cosmological constant by Einstein (?)

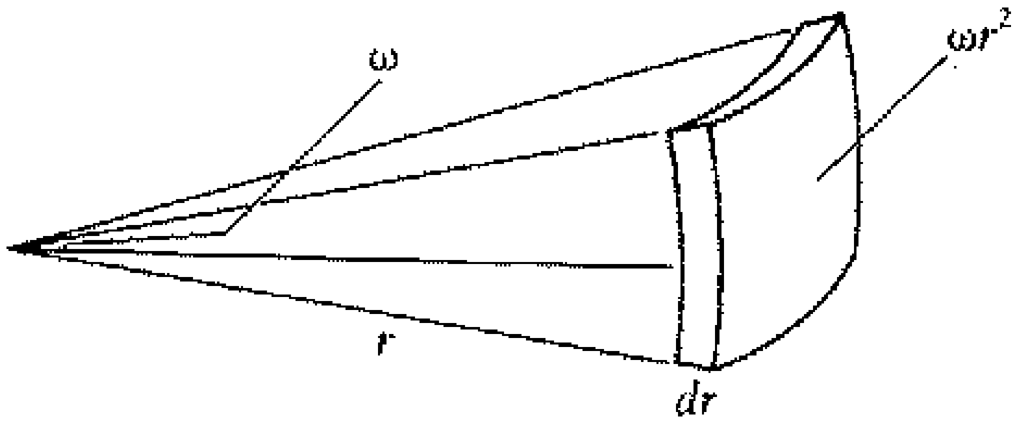
  $\longrightarrow$ to have solutions under the starting static hypothesis.

- early universe physics

  $\longrightarrow$ advocates unknown physics (inflation, ...)

Olbers paradox

Volume element for counts:



Olbers paradox

Homogeneous medium of stars:

$$l = \frac{L}{4\pi r^2}$$

Number of stars between r and $r + dr$:

$dN = n_* \times \omega \times r^2 \times dr$

Number of stars between l and $l + dl$:

$$dN = -\frac{1}{2} n_* \omega \left(\frac{L}{4\pi} \right)^{3/2} \frac{dl}{l^{5/2}}$$

Number counts:

$$N(> l) = \frac{1}{3} n_* \omega \left(\frac{L}{4\pi} \right)^{3/2} \frac{1}{l^{3/2}}$$

in term of magnitude ($m = -2.5 \log(l)$+cste)

$$\log(N(< m)) \propto 0.6m + \text{cste}$$

Olbers paradox

Integrated luminosity:

$$\phi = \int_0^{+\infty} \frac{dN}{dl} l dl$$

$l^{-3/2}$ makes the integral diverging!

Something wrong among:

- The universe is homogeneous
- Universe is static and eternal
- Geometry of space is Euclidian geometry

In retrospect, now that we have reasonably convincing evidence that the universe really is expanding, it is easy to find reasons why a static universe is problematic.

J.Peebles

Olbers paradox

Energetics considerations:

- Finite volume =>
- Finite number of stars =>
- Finite amount of energy is available

Olbers paradox

Timescale for energy exhaust:

Order of magnitude for the Sun:

$$L_\odot \sim 4\ 10^{33}\ \text{ erg/s}$$

$$M_\odot \sim 2\ 10^{33}\ \text{ g}$$

Efficiency of nuclear reactions $\epsilon \sim 0.007$

$$\tau \sim \frac{\epsilon M_\odot c^2}{L_\odot} = \frac{E_t}{\frac{dE_t}{dt}}$$

$$\sim 3\ 10^{18} s \sim 10^{11}\ \text{yr}$$

=> the universe cannot remain identical for ever!

Homogeneity

“The universe is homogeneous on large scale”

Einstein cosmological principle

Can and should be tested from observations.

$$\lim_{R \to \infty} \overline{\rho}(R) = cste$$

(necessary but not sufficient...)

From galaxies:

$$\log(N(m)) \propto 0.6m \; ?$$

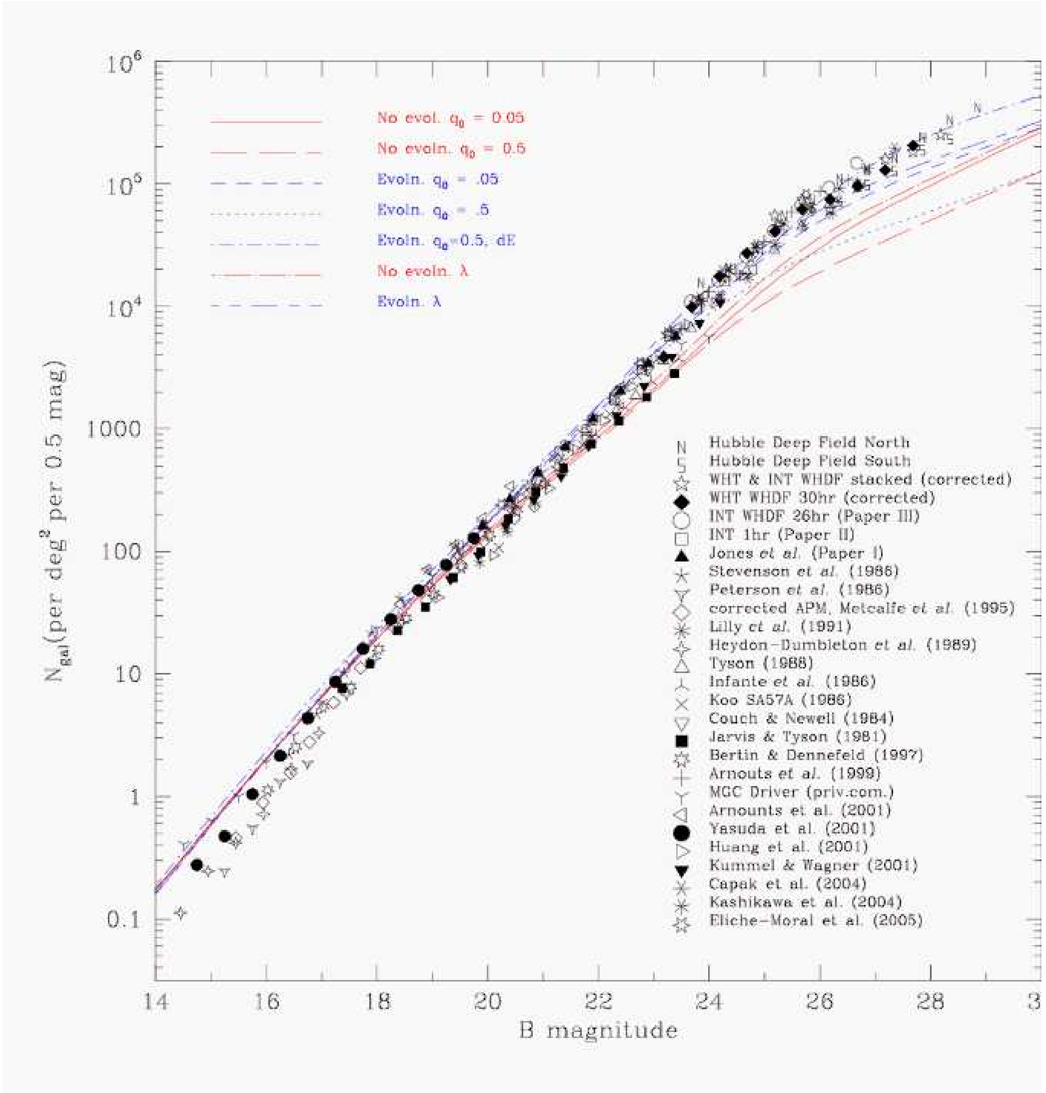
good indication.

Isotropy

+Copernic principle => homogeneity

Homogeneity

From galaxies number counts:



Geometry

- locally we can assign four coordinates to an event (x, y, z, t)
- this does not prejudge of the global *shape* of the universe: Plane ? Sphere ? Torrus ?

3D Spherical universe: start from 4D (x, y, z, u)

$$x^2 + y^2 + z^2 + u^2 = R^2$$

using spherical coordinates $r^2 = x^2 + y^2 + z^2$
from $dl^2 = dx^2 + dy^2 + dz^2 + du^2$ and $u^2 = R^2 - r^2$ one gets:

$$dl^2 = r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad + \quad \frac{dr^2}{1 - \left(\frac{r}{R}\right)^2}$$

General Geometry

$$dl^2 = r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad + \quad \frac{dr^2}{1 - \left(\frac{r}{R}\right)^2} \text{ (spherical)}$$

$$+ \quad dr^2 \text{ (flat)}$$

$$+ \quad \frac{dr^2}{1 + \left(\frac{r}{R}\right)^2} \text{ (hyperbolic)}$$

The Robertson-Walker line element: $r \longrightarrow \frac{r}{R}$

$$ds^2 = -c^2dt^2 + R(t)^2[r^2(d\theta^2 + \sin^2\theta d\phi^2) + \frac{dr^2}{1 - kr^2}]$$

with $k = -1, 0, +1$ accordingly to geometry.

General Geometry

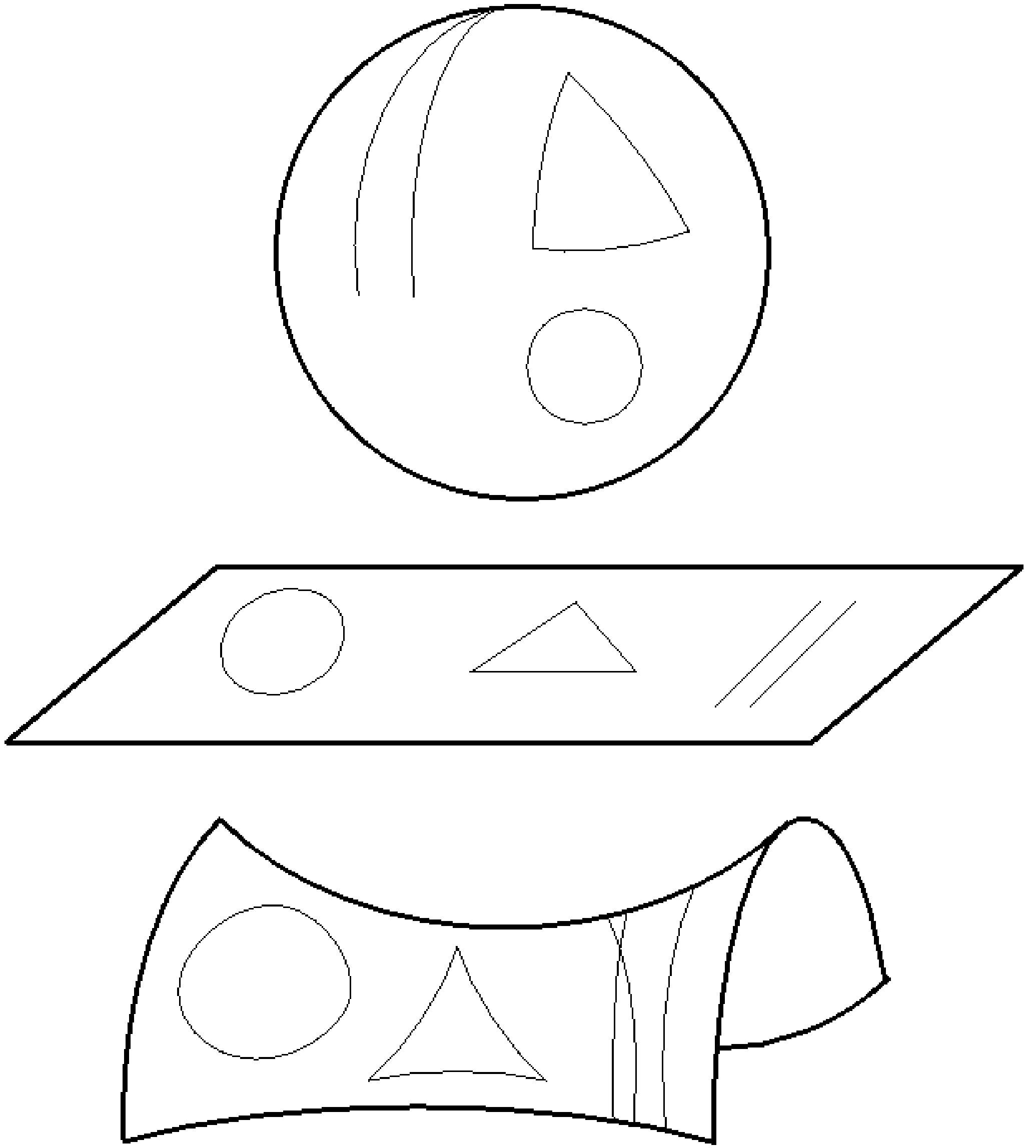
The Robertson-Walker line element: $r = rR_0$

$$ds^2 = -c^2 dt^2 + a(t)^2 [r^2 (d\theta^2 + \sin^2\theta d\phi^2) + \frac{dr^2}{1 - Kr^2}]$$

with $K = \frac{k}{R_0}$ and $a(t_0) = 1$.

General Geometry

Three possible geometries:



Topology

The local geometry of space (i.e. the value of k) does not prejudge of the global shape of space i.e. its topology.

-> The universe could be finite even with $k = 0, -1$.