Search for a Quantum Gravity Signature with HETE-2 GRBs with measured redshifts

J. Bolmont, J.-L. Atteia, A. Jacholkowska & F. Piron

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More details in astro-ph/0603725
submitted to ApJ
Overview

- Test of Lorentz Invariance in the frame of String Theory
- Use of 15 GRBs with measured redshifts detected by HETE-2
- Use of photon-tagged data
- Study of different energy bands
- Light curves analysed using Wavelets
- Result: limits on Quantum Gravity energy scale $E_{\text{QG}}$
Studies with GRBs...

- With GRB 021206
  - $E_{\text{QG}} > 1.8 \times 10^{17}$ GeV

- With 9 GRBs with known $z$ seen by BATSE and OSSE
  - $E_{\text{QG}} > 6 \times 10^{15}$ GeV

- With 35 GRBs with known $z$ seen by BATSE, HETE-2 and SWIFT
  - $E_{\text{QG}} > 9 \times 10^{15}$ GeV

- With 17 GRBs including 11 with known $z$ seen by INTEGRAL
  - R. Lamon et al., arXiv:0706.4039
  - $E_{\text{QG}} > 1.5 \times 10^{14}$ GeV, introducing a correction for intrinsic lags
Other sources...

- Any transient source can be used...
- Crab pulsar seen by EGRET
  - $E_{\gamma} > 1.8 \times 10^{15}$ GeV
- Flare of Mkn 421 seen by the Whipple Observatory
  - $E_{\gamma} > 6 \times 10^{16}$ GeV
- Flare of Mkn 501 seen by MAGIC
  - J. Albert et al., arXiv:0708.2889
  - $E_{\gamma} > 2.6 \times 10^{17}$ GeV
- See A. Sakharov’s talk this afternoon...
The Model
Dispersion relation

- Model based on String Theory
- Quantum fluctuations in the structure of space-time at the Planck scale
  \[ l \sim l_P \approx 10^{-33} \text{ cm} \quad E \sim M_P \approx 10^{19} \text{ GeV} \]
- J. Ellis et al., Beyond the Desert 99, Ringberg, June 1999 (gr-qc/9909085)
- Vacuum with a non-trivial refractive index
- The speed of photons change with their energy: \( v(E) = \frac{c}{n(E)} \)
- Dispersion relation with \( E/E_{QG} \) expansion:
  \[ c^2 p^2 = E^2 \left( 1 + \xi \frac{E}{E_{QG}} + O \left( \frac{E^2}{E_{QG}^2} \right) \right) \]
  so at the first order:
  \[ v(E) \approx c \left( 1 - \xi \frac{E}{E_{QG}} \right) \] with in the following \( \xi = 1 \)
We assume that two photons are emitted at the same time with energies $E_1$ and $E_2$ at redshift $z$.

The dispersion relation tells us that the photons travel with different speeds $v_1$ and $v_2$.

We need to express the time lag $\Delta t$ as a function of $z$ and $\Delta E = E_2 - E_1$. 

$E_2 > E_1$
Time lags & Cosmology

- Time-redshift relation:

\[ dt = -H_0^{-1} \frac{dz}{(1 + z) h(z)} \]

where

and

- When measured at redshift \( z \), a particle with velocity \( u \) travels a distance:

\[ dl = u \, dt = -H_0^{-1} \frac{u \, dz}{(1 + z) h(z)} \]

in a time \( dt \).

- At redshift 0, this distance is:

\[ dl_0 = -H_0^{-1} \frac{u \, dz}{h(z)} \]

\( h(z) = \sqrt{\Omega_\Lambda + \Omega_M (1 + z)^3} \)

\( \Omega_{tot} = \Omega_\Lambda + \Omega_M = 1 \)

\( \Omega_\Lambda = 0.7 \)

\( H_0 = 71 \text{ km s}^{-1} \text{ Mpc}^{-1} \)

\( \Omega_t = \Omega_\Lambda + \Omega_M = 1 \)

\( \Omega_\Lambda = 0.7 \)
Time lags & Cosmology

- Two particles with velocities different by $\Delta u$ travel distances different by

$$\Delta L = H_0^{-1} \frac{\Delta u \, dz}{h(z)}$$

- Using the dispersion relation, we have

$$\Delta u = -c \frac{\Delta E (1 + z)}{E_{QG}}$$

- Then, we get

$$\Delta t = H_0^{-1} \frac{\Delta E}{E_{QG}} \int_0^z \frac{(1 + z) \, dz}{h(z)}$$

- In the following, we confront the data and the relation

$$\langle \Delta t \rangle = a \, K_l(z) + b \, (1 + z)$$

with

$$K_l(z) = \int_0^z \frac{(1 + z) \, dz}{h(z)}$$
This definition is different from the one used in previous studies:

\[ K_l(z) = \int_0^z \frac{(1 + z) \, dz}{h(z)} \]
How to test this model?

- Gamma-Ray Bursts are well suited for this kind of study:
  - Transient sources
  - Bright
  - At cosmological distances
  - Wide energy range (0.1 MeV - 10 GeV)

- Problems:
  - Emission mechanisms at 100 keV and ~1 GeV may be different
  - Intrinsic time lags (source effects)
The Data
**HETE-2**
2000-2006

- Goal: fast localisation of GRBs (~250 seen, ~80 localised, ~20 with z)
- Large energy range: 0.5-600 keV (soft & medium X-rays + γ-rays)
- **WXM, SXC, FREGATE**
- **FREGATE:**
  - Energy range: 6-400 keV
  - Effective area: 160 cm$^2$
  - Field of view: 70°
  - Time resolution: 6.4 µs
  - Spectral resolution at 122 keV: ~12%
- 15 GRBs with measured redshift AND photon tagged data available...
The data

- 15 GRBs
- Kl(z) injective
- 0.16 < z < 3.37
- 0.07 s < T90 < 89.3 s
- 10 GRBs with L > 8×10^{51} erg/s

<table>
<thead>
<tr>
<th>GRB</th>
<th>z</th>
<th>Kl</th>
<th>T90 (s) *</th>
<th>L_{51} (erg/s)</th>
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<tbody>
<tr>
<td>GRB 050709</td>
<td>0.16</td>
<td>0.17</td>
<td>0.07</td>
<td>-</td>
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<td>0.41</td>
<td>0.44</td>
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<td>89.3</td>
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<td>18.6</td>
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<tr>
<td>GRB 021004</td>
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<td>GRB 020124</td>
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<td>3.32</td>
<td>46.4</td>
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<tr>
<td>GRB 060526</td>
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<td>3.34</td>
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<tr>
<td>GRB 030323</td>
<td>3.37</td>
<td>3.47</td>
<td>27.8</td>
<td>11.92</td>
</tr>
</tbody>
</table>

*T90 given for the energy range 30-400 keV.
The data
The Method
Our method

- Inspired by

- For each burst, the time interval to be studied between start and end of burst is determined
- The two energy bands for the time lag calculations are chosen
- De-noising of the light curves by a Discrete Wavelet Transform
- Search for rapid transitions in the light curves using a Continuous Wavelet Transform
- Association of extrema in pairs
- For each GRB, the average time lag $\langle \Delta t \rangle$ is computed
- Study of $\langle \Delta t \rangle$ as a function of $z$
Time interval

- Noise is studied outside of the burst region
- A projection gives the background level and the variance of the noise
- Light curve in 6-400 keV is denoised and the background is subtracted
- Signal is studied where it exceeds $1\sigma_{\text{noise}}$
- A limit of $0.5\sigma_{\text{noise}}$ had to be chosen for 030323, 030429 and 060526
### Energy bands

- **14 scenarios**
- **Scenario = choice of E.B. 1 and E.B. 2 (**)**
- **For each GRB, 14 values of the mean $<\Delta E>$**

<table>
<thead>
<tr>
<th>Scenario</th>
<th>E. B. 1</th>
<th>E. B. 2</th>
<th>Mean $&lt;\Delta E&gt;$ **</th>
</tr>
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<tbody>
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<td>#1</td>
<td>20-35</td>
<td>60-350</td>
<td>117.6</td>
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<td>#2</td>
<td>8-30</td>
<td>60-350</td>
<td>127.2</td>
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<td>#3</td>
<td>8-20</td>
<td>60-350</td>
<td>130.2</td>
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<td>8-20</td>
<td>30-350</td>
<td>85.0</td>
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<td>8-30</td>
<td>30-350</td>
<td>82.0</td>
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<td>#6</td>
<td>8-20</td>
<td>40-350</td>
<td>102.8</td>
</tr>
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<td>#7</td>
<td>8-30</td>
<td>40-350</td>
<td>99.8</td>
</tr>
<tr>
<td>#8</td>
<td>8-40</td>
<td>40-350</td>
<td>97.9</td>
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<td>#9</td>
<td>20-35</td>
<td>40-350</td>
<td>90.1</td>
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<td>#10</td>
<td>8-20</td>
<td>50-350</td>
<td>116.9</td>
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<tr>
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<td>8-30</td>
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<tr>
<td>#14</td>
<td>20-35</td>
<td>50-350</td>
<td>104.2</td>
</tr>
</tbody>
</table>

* E. B. = Energy Band  
** Average for all GRBs  
All energies are given in keV
Energy bands

![Graph showing energy bands with model parameters: Model: cutoffpl, \(E_0: 100.200\) keV, \(\alpha = 0.0670\), Norm: 52.460, \(\chi^2: 133.742\), Reduced-\(\chi^2: 1.274\).]
De-noising

WaveLab, D. Donoho et al., http://www-stat.stanford.edu/~wavelab

- Discrete Wavelet Transform
- Symmlet-10
- WaveLab v802
  - Input: light curve
  - Output: de-noised light curve
Localisation of Extrema

LastWave, E. Bacry, http://www.cmap.polytechnique.fr/~bacry/LastWave/

- Continuous Wavelet Transform
- Mexican-hat wavelet
- LastWave 203

- Input: de-noised light curve
- Output: list of extrema with $t$, $\alpha$, $\delta\alpha$

More about wavelets:
S. Mallat, A Wavelet Tour of Signal Processing Academic Press, San Diego
Selection of extrema

- For each extrema, we know:
  - The position $t$
  - The Lipschitz coefficient $\alpha$
  - The error $\delta \alpha$

- To reject fake extrema, we first select extrema with

\[
\left| \frac{\Delta f}{\Delta t} (t = t_0) \right| \leq 0.2
\]
Selection of pairs

- Pair = (extrema in E.B. 1, extrema in E.B. 2)
- We use the variables

\[
\begin{align*}
\Delta t &= t_2 - t_1 \\
|\Delta \alpha| &= |\alpha_2 - \alpha_1| \\
\delta(\Delta \alpha) &= \sqrt{\delta \alpha_2^2 + \delta \alpha_1^2}
\end{align*}
\]

- Studying the distributions, we use the cuts

\[
\begin{align*}
|\Delta t| &< 150 \text{ ms} \\
\Delta \alpha &< 0.4 \\
\delta(\Delta \alpha) &< 0.045
\end{align*}
\]
Results
Mean Time Lag vs. $z$

$< \Delta t > = a K_l(z) + b (1 + z)$

No variation of $a$ above $\pm 3\sigma$
\( \chi^2 \) function

- We define the following likelihood function
  \[
  L = \exp \left( -\frac{\chi^2(M)}{2} \right)
  \]

  with

  \[
  \chi^2(M) = \sum_{i=1}^{N_{\text{GRB}}} \frac{(\Delta t_i - \tilde{b}(1 + z_i) - a_i(M) K_{li})^2}{\sigma_i^2 + \sigma_{\tilde{b}}^2}
  \]

  \( N_{\text{GRB}} \leq 15 \) and

  \[
  a_i(M) = \frac{1}{H_0} \frac{\Delta < E >_i}{M}
  \]

- \( \Delta < E > = < E >_2 - < E >_1 \) is calculated for each GRB

- Universality of intrinsic source time lags is assumed and we define

  \[
  \tilde{b} = \frac{\sum_k w_k b_k}{\sum_k w_k}, \quad \sigma_{\tilde{b}} = \frac{1}{\sqrt{\sum_k w_k}} \quad \text{with} \quad w_k = 1/\sigma_k
  \]
$a$ and $b$ are correlated
$\chi^2$: Maxima only

![Graph showing the $\chi^2$ distribution for different scenarios and mass ranges.]

- $E_{QG} > 3.2 \times 10^{15} \text{ GeV}$
$\chi^2$: Minima only

- Scenario
  - 20-35 keV - 60-350 keV
  - 8-30 keV - 60-350 keV
  - 8-20 keV - 60-350 keV
  - 8-20 keV - 30-350 keV
  - 8-30 keV - 30-350 keV
  - 8-30 keV - 40-350 keV
  - 8-30 keV - 40-350 keV
  - 8-40 keV - 40-350 keV
  - 8-50 keV - 40-350 keV
  - 20-35 keV - 40-350 keV
  - 8-20 keV - 50-350 keV
  - 8-30 keV - 50-350 keV
  - 8-40 keV - 50-350 keV
  - 8-50 keV - 50-350 keV
  - 20-35 keV - 50-350 keV

- $M_{\text{min}}$ (GeV)
  - $10^{14}$
  - $10^{15}$
  - $10^{16}$
  - $10^{17}$
  - $10^{18}$

- $E_{\text{QG}} > 7.5 \times 10^{14}$ GeV
$\chi^2$: All extrema

95% CL
Maxima & Minima
Scenario
20-35 keV  -  60-350 keV
8-30 keV  -  60-350 keV
8-20 keV  -  60-350 keV
8-20 keV  -  30-350 keV
8-30 keV  -  30-350 keV
8-20 keV  -  40-350 keV
8-30 keV  -  40-350 keV
8-20 keV  -  50-350 keV
8-30 keV  -  50-350 keV
8-40 keV  -  50-350 keV
20-35 keV  -  50-350 keV

$E_{QG} > 2.0 \times 10^{15}$ GeV
Limits

- Limits at 95% CL
- All limits are in $10^{14}$-10$^{15}$ GeV
- Best limits for scenario #3
  - $<\Delta E> \sim 130$ keV
- No clear correlation between the limit and $<\Delta E>$

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Minima</th>
<th>Maxima</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>$5.9 \times 10^{14}$</td>
<td>$5.8 \times 10^{14}$</td>
<td>$5.7 \times 10^{14}$</td>
</tr>
<tr>
<td>#2</td>
<td>$5.8 \times 10^{14}$</td>
<td>$7.8 \times 10^{14}$</td>
<td>$9.0 \times 10^{14}$</td>
</tr>
<tr>
<td>#3</td>
<td>$7.5 \times 10^{14}$</td>
<td>$3.2 \times 10^{15}$</td>
<td>$2.0 \times 10^{15}$</td>
</tr>
<tr>
<td>#4</td>
<td>$3.6 \times 10^{14}$</td>
<td>$5.8 \times 10^{14}$</td>
<td>$6.3 \times 10^{14}$</td>
</tr>
<tr>
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<td>$4.6 \times 10^{14}$</td>
<td>$4.6 \times 10^{14}$</td>
</tr>
<tr>
<td>#6</td>
<td>$4.7 \times 10^{14}$</td>
<td>$1.8 \times 10^{15}$</td>
<td>$1.3 \times 10^{15}$</td>
</tr>
<tr>
<td>#7</td>
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<td>$1.7 \times 10^{15}$</td>
<td>$4.5 \times 10^{14}$</td>
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<td>$1.8 \times 10^{15}$</td>
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<td>$1.2 \times 10^{15}$</td>
<td>$5.2 \times 10^{14}$</td>
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<td>$2.3 \times 10^{14}$</td>
<td>$7.7 \times 10^{14}$</td>
<td>$6.8 \times 10^{14}$</td>
</tr>
</tbody>
</table>

All limits are given in GeV
Systematics

Possible sources of systematic errors:

- CWT & DWT
- Derivative
- Association in pairs
- Study with different
  - Wavelet functions
  - Decomposition levels for the DWT
  - Cut values for the derivative
  - Cut values for pair association
- No significant change of the limits!
- Lower values due to decreased statistical power...

\[
\left| \frac{\Delta f}{\Delta t} (t = t_0) \right| \leq 0.1, 0.2, 0.3
\]

\[
\left\{ \begin{array}{l}
\Delta \alpha < 0.2 \\
\delta(\Delta \alpha) < 0.045 \\
\Delta \alpha < 0.4 \\
\delta(\Delta \alpha) < 0.02
\end{array} \right.
\]
Intrinsic lags

- GRBs suffer from intrinsic lags!
- At higher energies, emission peaks
  - are shorter
  - arrive earlier

⇒ Necessity to use several sources with different z...
Intrinsic lags

- Strong correlation between lags and peak luminosity
- At low redshift, faint & bright bursts
- At high redshift, only bright bursts
- Test with a limited sample with $L > 8 \times 10^{51}$ erg/s (10 GRBs)

$$L_{53} \approx 1.3 \times \left( \frac{\tau}{0.01 \text{ s}} \right)^{-1.14}$$
$L_{51} > 8$ erg/s: Maxima only

$E_{QG} > 8.7 \times 10^{14}$ GeV
\( L_{51} > 8 \) erg/s: Minima only

\[ E_{QG} > 8.4 \times 10^{14} \text{ GeV} \]
L_{51} > 8 \text{ erg/s: All extrema}

\[ \Delta \chi^2 / \text{ndf} \]

95\% CL
Maxima & Minima

E_{QG} > 3.9 \times 10^{15} \text{ GeV}
Conclusions
Values (a = 0, b = 0) cannot be excluded

No big difference between limits for minima and maxima

All limits in the $10^{14}$-$10^{16}$ GeV range

Full sample:

- Preferred value of $E_{QG}$ for minima
- No preferred value for maxima

$L_{51} > 8$ erg/s (10 GRBs):

- Preferred value of $E_{QG}$ for maxima
- No preferred value for minima

Final result: full sample, considering maxima & minima

$E_{QG} > 2.0 \times 10^{15}$ GeV
What next?

Three factors to improve the limits:
- Larger GRB sample with measured $z$
- Detectors with better time resolution
- Better understanding of emission mechanisms and of source effects